ACCURATE CALCULATION OF THE MODES OF THE CIRCULAR MULTIRIDGE WAVEGUIDE

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ABSTRACT

Circular multiridge waveguides (CMRW) have been recently considered for their promise in application to tuningless dual mode filters.
We propose an accurate method for characterizing such structures that is based on a combination of the Boundary Integral Method, in a form leading to a standard eigenvalue problem, with a proper choice of the current distributions on the ridges, which takes into account the edge effects. The resulting code permits to calculate very accurately several tens of modes by inverting only once a reasonably small matrix.

INTRODUCTION

The realization of tuningless dual mode filters requires an appropriate choice of the coupling sections that must be precisely manufactured [1]. Multiridge circular sections seem to be an ideal candidate for this purpose and for this reason they have been recently examined by several researchers [2], [4], [3], [5], to cite a few among many others.
After careful consideration, we have come to the conclusion that the most effective method of analysis for this class of waveguides known to us appears a further modification of the *Boundary Integral Method (BIM)* in the formulation proposed by Conciauro et al. Unlike many other semi-analytical methods, which generate the eigenvalues by a search of the zeros of a determinant, *the unknown eigenvalues arise by the solution of a classical eigenvalue problem.* This fact makes the approach very suitable for problems requiring the calculation of several modes. Moreover, we have improved the accuracy of the method by expanding the electric currents on the ridges into a set of orthonormal polynomials weighted by the correct edge conditions, resulting from the exact solution of Maxwell's equations around the edges [8]. Although such a choice guarantees higher accuracy, the evaluation of the integrals of the Green's function and the expanding functions is now more complex, because of the singular behaviour of the latter near the edges. We have introduced therefore a suitable change of variables that eliminates the singularities. We have also extracted the singularities of the static part of the Green's function and calculated their integrals analytically. The integrals of the functions thus regularized have then computed numerically.

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OUTLINE OF THE METHOD

The dyadic Green's function relating the unknown currents to the electric field into a circular waveguide is expressed as the sum of a static part $\mathbf{G}_s$ and a dynamic one, the latter being a sum of a solenoidal part $\mathbf{G}_d$ and an irrotational part $\mathbf{G}_i$. The static term is known analitically, while the dynamic solenoidal is given by a summation of two very rapidly converging series [6]:

$$\mathbf{G}_d = \mathbf{zz} \quad \sum_m \quad \frac{k^2}{{k_m}^2({k_m}^2 - k^2)}\psi_m(\mathbf{r})\psi_m(\mathbf{r}') + \sum_m \quad \frac{k^2}{{k_m}'^2({k_m}'^2 - k^2)}\mathbf{e}_m^H(\mathbf{r})\mathbf{e}_m^H(\mathbf{r}') \quad (1)$$

Where $\mathbf{z}\,\psi_m(\mathbf{r})$ and $\mathbf{e}_m(\mathbf{r})$ are respectively E and H eigenvectors of the circular housing and k_m and k_m' the corresponding eigenvalues. The irrotational

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part is also known analitically :

$$\mathbf{G}_i = -\frac{1}{k^2}\nabla\nabla g(\mathbf{r},\mathbf{r}') \quad (2)$$

The reader can be found the complete expression of the Green's function in[5]. A fundamental point is that the eigenvalue k of the structure under exam only appears in $\mathbf{G}_d$ and $\mathbf{G}_i$, $\mathbf{G}_s$ containing only a static contribution. The eigenvalue is determined by enforcing the vanishing of the tangential electric field over the contour of each ridge:

$$< \mathbf{G}_s, \mathbf{J} > + < \mathbf{G}_i, \mathbf{J} > + < \mathbf{G}_d, \mathbf{J} >= 0 \quad (3)$$

For istance, in the E case, we obtain:

$$< \mathbf{G}_s, \mathbf{J} > + \sum_m \frac{a_m}{{k_m}^2}\psi_m(\mathbf{r}) = 0 \quad (4)$$

$$\frac{{k_m}^2 - k^2}{k^2} a_m =< \mathbf{z}\psi_m(\mathbf{r}), \mathbf{J} > \quad (5)$$

The above system is discretized according to the Galerkin method, by expanding the unknown current $\mathbf{J}$ into a set of N basis functions and truncating the series appearing in (5) after the M-th mode. The resulting eigenvalue problem assumes the form of a standard $(N+M)\times(N+M)$ linear system:

$$(\mathbf{A} + k^2\mathbf{B})\mathbf{X} = \mathbf{0} \quad (6)$$

The first N components of the eigenvector $\mathbf{X}$ are the Fourier coefficients of the current associated to the eigenvalue k, with respect to the set chosen, while the remaining M components are the amplitudes a_j of the corresponding field with respect to the modes of housing. For the sake of brevity we will detail only the discretization in the single ridge case (Fig.1).

TM modes

In order to calculate the TM modes, the unknown current has been discretized by setting $J_z = w_i \sum_k^{N_i} b_k f_k$, $\sum_{i=1}^{3} N_i = N$ where:

$$\begin{aligned} f_k^1(x) = f_k^3 &= \frac{1}{\sqrt{a-R}}\frac{1}{N_{gk}} G_k(\frac{4}{3},\frac{5}{3},x) \\ w_1(x) = w_3 &= x^{2/3}(1-x)^{-1/3},\ x = \frac{a-r}{a-R} \end{aligned} \quad (7)$$

$$\begin{aligned} f_k^2(y) &= \frac{1}{\sqrt{R\Delta\phi}}\frac{1}{N_{ck}} C_k^{1/6}(y) \\ w_2(x) &= (1-y^2)^{-1/3},\ y = \frac{\phi - \Phi - \Delta\phi}{\Delta\phi} \end{aligned} \quad (8)$$

The index i denotes the i-th segment defining the ridge, G_k and C_k are respectively Jacobi and Gegenbauer polynomials, while N_{gk} N_{ck} are the corresponding normalization constants [7]. The use of the above functions guarantees the best accuracy in the description of the unknown currents, since their behaviour around the edges is implicitly taken into account by the weight functions w_i. Unfortunately, the resulting overlapping integrals are difficult to evaluate analytically and, moreover, they are not easy to evaluate numerically because of the singularities. In order to overcome such difficulties, we have introduced a change of variable that eliminates the integrable singularities of the weight functions. This transformation is given by:

$$x = \sin^2 u \quad (9)$$

$$y = \sin v \quad (10)$$

After the transformation, the products between weight functions and differentials (dx, dy) become:

$$\begin{aligned} w_1(x(u))\,dx &= 2\sin^{7/3} u \cos^{1/3} u\,du \\ w_2(y(v))\,dy &= \cos^{1/3} v\,dv \end{aligned} \quad (11)$$

TE modes

This case is rather more difficult than the previous one. That is due to the following reasons:

1. The problem is vectorial.
2. The form of the static part of the Green's function is much more complicated than that of the TM case [5].
3. The unknown currents, given by H_z, must be expanded into a set able to describe two possible different behaviours near the edges [8]:
   (a) H_z proportional to $J_0(1-\frac{a-r}{a-R})$.
   (b) H_z proportional to $J_{2/3}(1-\frac{a-r}{a-R})$.
4. the first derivative of the expanding functions are also involved in the calculation of the static terms of (6).

In addition, some attention must be payed to the current about the edge connecting the arc and the ridge, where the current does not vanish while its derivative does. The unknown longitudinal magnetic field H_z has been expressed in terms of Gegenbauer polynomials of order 7/6, $C_x^{7/6}(z)$ weighted by a function $w = (1 - z^2)^{2/3}$, where $z = x$ or y as defined in (7) and (8), depending upon the discretization subregion, plus a constant in order to describe the behaviour near the inner edge discussed above. The log singularities contained in the static part of the Green's function were isolated in a standard way and integrated analytically.

RESULTS

We tested the method comparing our results with those obtained by Vahldieck [4] for the first eigenvalues of single and multi ridge waveguides, obtaining results that are indistinguishable from this. In order to show the accuracy of the technique and of the discretization, we computed the even and odd modes of a single ridge penetrating completely into the circular waveguide, as shown in Fig. 2. In that case, in fact, the azimuthal boundary of the ridge collapses down to a point, making quite hard the computation of the integrals, which involve the singularity of the Green function; moreover, accuracy depends strongly on the number of the housing modes as well as on dimensions of the ridge. Fig. 3- 6 show the excellent accuracy achieved in the determination of the first 90 even and odd TM and TE modes in the above critical case, for a ridge angle of 20°
The example was perfomed by taking 9 basis functions over regions 1 and 3, 1 function over region 2 and considering 190 modes of the housings. The cpu time required was about 120 sec on a Digital α series 300 for the TM modes and about 600 sec for the TE ones. A computation of 200 TM eigenvalues for the triple ridge required 400 sec, taking 19 basis polynomials for each ridge.

CONCLUSIONS

We present a truly accurate and efficient technique for the calculation of many modes of the multiridge circular waveguide, based on a modified Boundary Integral Method, in a form leading to an eigenvalue algebraic problem, and on a proper choice of the basis functions taking into accout the correct edge conditions. The efficiency of the algorithm is enforced by a suitable transformation of the integrand variables so as to eliminate the singularities of the expanding functions. The accuracy of the method is tested by analyzing hundred of modes of a sectorial waveguide.

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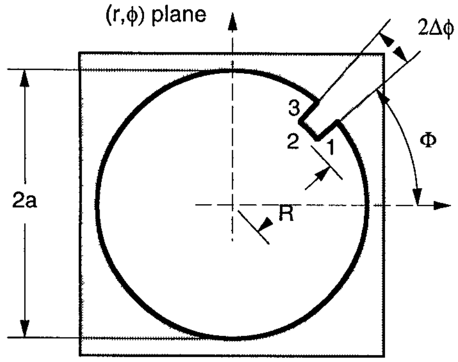

Fig. 1. Cross section of a single ridge guide with the coordinate systems

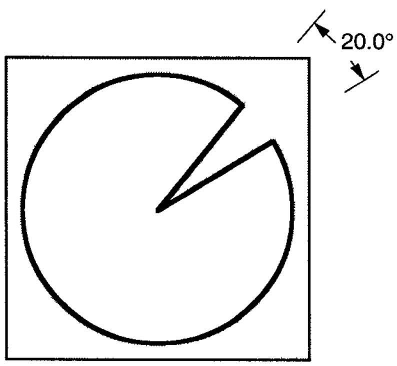

Fig. 2. Cross section of the single ridge guide fully penetrating into the circular waveguide considered in the numerical example

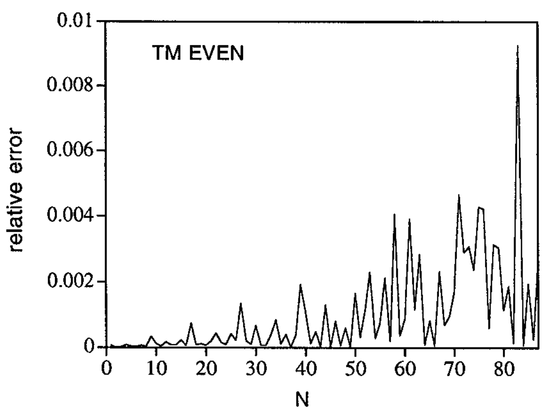

Fig. 3. Relative error involved in the calculation of the first 90 even TM eigenvalues for a single ridge fully penetrating the circular waveguide

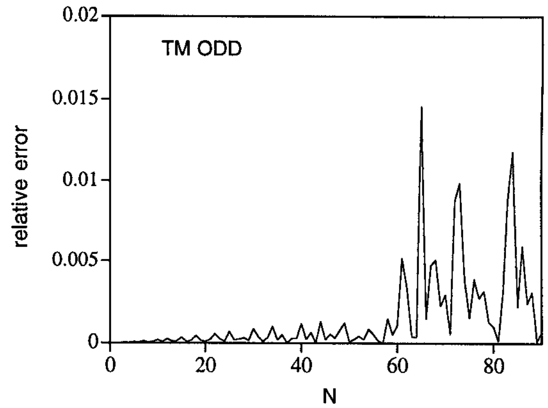

Fig. 4. Relative error involved in the calculation of the first 90 odd TM eigenvalues for a single ridge fully penetrating the circular waveguide

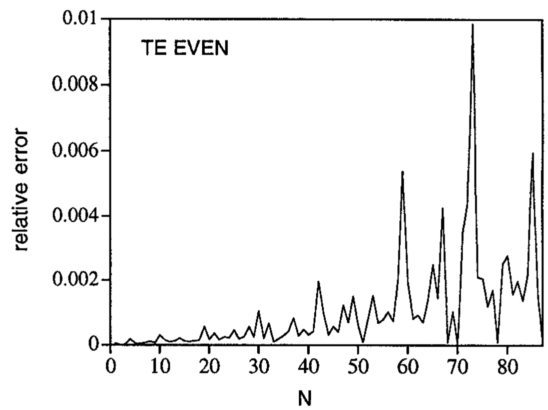

Fig. 5. Relative error involved in the calculation of the first 90 even TE eigenvalues for a single ridge fully penetrating the circular waveguide

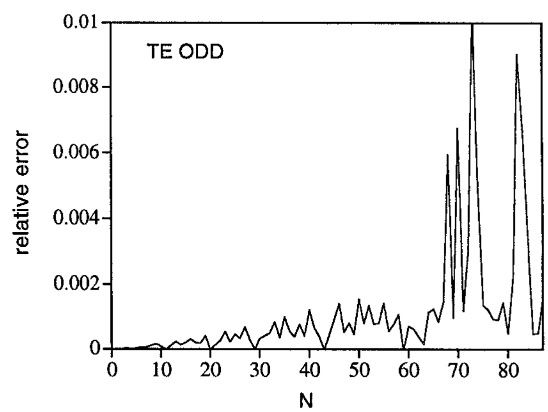

Fig. 6. Relative error involved in the calculation of the first 90 odd TE eigenvalues for a single ridge fully penetrating the circular waveguide

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